Fast Forecasting of Time Series Data

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March 10, 2001

Abstract

A time sequence is an ordered set of measurements of some observable over time. When the underlying dynamical system isn't fully understood, observed regularities in the time series can provide insights into the inner working of the system. Given a time series from the past, the aim of a prediction algorithm is to forecast the expected future of the time series by drawing analogies from previous behaviour. I propose to work on methods for fast and efficient prediction of time sequences. Time sequences show up in a variety of natural and artificial systems, often under circumstances where the precise equations for the system are unknown. Thus, this area has wide applicability.

My approach uses a technique called *delay coordinate embedding*, in which the time series is broken up into equal-sized chunks which represent the internal state of the system. Whenever a prediction needs to be made, the latest chunk is found, and the chunks "nearest" to it in "chunk space" are retrieved. By observing how the time series developed in each of these cases, its expected course in the current circumstances can be predicted. This involves several issues:

- Deciding the size of the chunks and their method of storage in a database
- Method for retrieving the best k-Nearest-Neighbour chunks
- Deciding the number k of nearest neighbours to be used, and a method of interpolation for combining the results from the k nearest chunks found
- Scalability to large databases

I will focus on developing formal methods (as opposed to ad hoc methods) for these problems.

As of now, there is already a prototype system which allows predictions on stock price data and on some transportation data. Chunk sizes were chosen on an ad hoc basis, and although the system is designed to be scalable, it has not been tested on such datasets. This project should result in the achievement of these aims.

1 Introduction

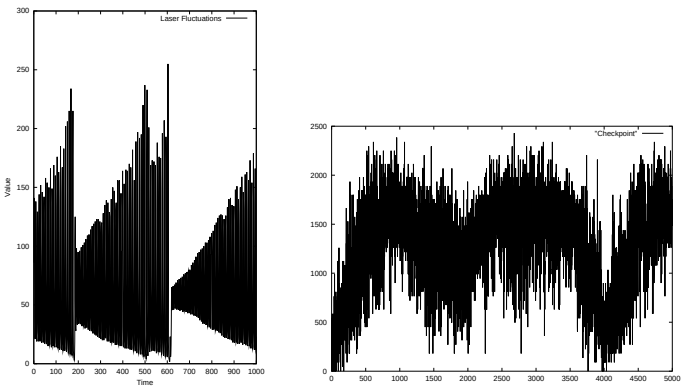
1.1 Problem Definition

Scientific research on physical systems is often based on the measurement of observables over time. Such data is the basis of characterization of the system and prediction of its future behaviour. In many real-world cases, the underlying equations for the system are unknown. Under such circumstances, regularities in the past time series have to be used as guides to understanding the system and forecasting the future.

Formally, the terminology involved is defined as follows:

- *Dynamical System:* This is a physical system which evolves over time.
- *System State:* This is the underlying state of the dynamical system under consideration, and is an unknown quantity.
- *Observable:* This is a function of the system state which can be be measured directly, or can be calculated from other measurements. If the system returns to the same state later, the observable will have the same value.
- *Time Series:* This is a list of values taken by the observable over time.

Given the time series data over some period, the problem is to predict the future evolution of the system. Several methods have been proposed, but many are slow or suffer from the problem of too many free parameters. My approach, called "Delay Coordinate Embedding" avoids this, but currently, no formal methods exist for setting the parameters in this method. This leads to arbitrariness, and unreplicable results. Solving this will be the thrust of my work. Some examples time series are shown in Figure 1.



(a) (b)

Figure 1: *Example time series*: Laser Fluctuations(a) and California Transportation Data(b)

1.2 Applications

The following gives a snapshot of the areas where time series have been shown up[4, 9]:

- *Physics:* Measurements of laser fluctuations.
- *Biology:* Physiological data, such as heart rate, blood oxygen concentration, and EEG state, of patients with sleep apnea.
- *Economics:* High-frequency currency exchange rate data, and daily stock quote data.
- *Astrophysics:* Intensity variations of white dwarf stars, prediction of solar activity.
- *Geophysics:* Measurements of magnetic storm data.
- *Transportation:* Highway usage-over-time data.

1.3 Deliverables

The final product will be an automated black-box, with a time series as input and an efficient prediction system as output. Internal parameters needed by the system will be set without resorting to thresholding/ bucketization, which introduce adhocness into the system. The prediction system will be generated based on these parameters. There will also be a validation system, which will be able to quantify the performance of the system in terms of computation costs and efficiency.

2 Related Work

Time-series prediction is a well-studied problem. Prior to the 1920s, forecasting was carried out by extrapolating the series through a global fit in the time domain. Then came the linear time series models, (such as the ARMA models), which attempted to describe the series as some linear function of internal and external variables. These models assume linearity and time invariance(ie., unchanging model coefficients over time) of the system. Reference texts for this are [6, 17]. They had some success, but failed to model non-linearities which often arise in real-world data.

Then, several non-linear approaches were developed. One such method is called the *delay coordinate embedding* method, which will be described in Section 3.1. Another approach has been through the use of Artificial Neural Networks [26, 27, 14, 25, 24]. Here, ANNs are modified to incorporate past values of time series. [24] describes a method where each network weight and neuron input is a vector encoding past time series values. Another network structure for time series prediction is called the *Time-Delay Neural Network* [13, 23]. In these too, embedded time delays are incorporated into the network structure. A taxonomy of neural network structures for time-series processing is provided in [16]. But all these methods suffer from the traditional problems associated with neural networks: long training times and large number of parameters.

Hidden Markov Models(HMMs) have also been used [10]. There is a similarity between these models and the Delay-Coordinate Embedding technique. The notion of a hidden state in the former is similar to that of a *reconstructed state space* used in the latter technique (refer Section 3.1). Discrete HMMs are not suitable for problems involving continuous data, so, a modified class of models called mixed-state HMMs are used. But, either these models do have non-linearities or they make somewhat strong assumptions. In either case, the Delay Coordinate Embedding approach seems to lead to a more natural problem description.

Another method is called the "Method of Analogues" [12]. This method takes a vector of the last p values of the time series (p being determined manually), and finds, say, the three best matches to this series from the entire time series history. These are the three nearest neighbours in p-dimensional space. Then, Singular Value Decomposition is used to fit a linear function through these nearest neighbours, and this function is used for prediction purposes. This approach is simple and has few free parameters, but works only for low-dimensional chaotic attractors, and only for predictions over a short period of time.

3 Approach

3.1 Background: Delay Coordinate Embedding

The *state* of a deterministic system carries all the information necessary for predicting the behaviour of the system. Under minimal requirements on the smoothness of the underlying equations, the existence and uniqueness of the *state* holds. Thus, at any moment, the system has a state, and if a state is ever repeated later, then the future behaviour of the system will also be the same, within limits placed by noise. Thus, the internal system state can provide information for prediction.

Let the observation sequence be represented by the series x_t, which gives the value of the time series at time t. Let us define the following vector:

$$\mathbf{b} = [\mathbf{x_t}, \mathbf{x_{t-\tau}}, \mathbf{x_{t-2\tau}}, \ldots, \mathbf{x_{t-(m-1)\tau}}] \tag{1}$$

Here, τ is some real number greater than zero, and m is any integer greater than zero. The vector $\mathbf{b}$ is called a *delay coordinate vector* because its terms are the time-delayed data values from the time series. The technique of delay coordinate embedding is based on certain results [22, 20] which state that any structure that exists in the state space is carried over faithfully into the delay-coordinate-vector space, and that it is a one-to-one transformation. Thus, properties of the state space, such as equilibrium, periodicity, Lyapunov exponents (for measuring the degree of chaos) can all be observed in delay-coordinate space. Of course, characterizing every possible state would require infinite data, so we typically can observe only a portion of the entire state space. Still, the delay coordinate vector can be a useful representation of the internal state of the dynamical

system. If, at two instances along the time series, the delay coordinate vectors are same (or very similar), then the data values following the two instances should be the same (or very similar). There is predictive power in finding this one-to-one map.

Hence, the problem of time series prediction is broken into two problems:

1. Finding the right length of the vector $\mathbf{b}$
2. Prediction of future values given the current delay coordinate vector

The first problem remains an art. For solving the second problem, the k-Nearest Neighbours $\mathbf{b}_1, \ldots, \mathbf{b}_k$ of the current state vector $\mathbf{b}$ are found, and the time-series values immediately succeeding these k instances (lets call them $p_1, \ldots, p_k$) are used to forecast the value after $\mathbf{b}$. One method is to just average over $p_1, \ldots, p_k$; another is to weight the p_i values by the exponential of the negative of the squared distance between $\mathbf{b}_i$ and $\mathbf{b}$. One particular implementation of this approach to time series prediction is given in [21]. Again, the determination of k is ad hoc.

3.2 Proposed Work

My proposed work is based on the delay-coordinate-embedding approach. One advantage of this approach is its firm mathematical basis [22, 20]. Again, in methods like ANNs, the number of free parameters has to be set a priori, which leads to the bias-variance dilemma. Large number of parameters also lead to long training times. Delay coordinate embedding avoids all these problems, and may lead to more scalable algorithms. The assumption of linearity is not made, which makes it usable for long-term predictions too. But there are problems with current implementations of the Delay Coordinate Embedding approach. The parameters needed by this approach, such as lag length , and number of nearest neighbours are set manually, which may lead to differences in results obtained by different experimenters. The thrust of my research will be on formalizing methods for setting these parameters, without the use of thresholding/bucketization techniques, which introduce arbitrariness into the system. Thus, we shall have a black box, which, given a time series, will generate a similarity search system based on this approach, with all parameters set automatically. I plan to use concepts borrowed from several fields: chaos theory, fractal theory and spatial databases. These ideas should help in fast parameter setting, even for large datasets. Also, the generated similarity search system will be optimized for that particular dataset, leading to fast predictions.

3.2.1 Using Chaos

Recent times have witnessed an upsurge in the application of chaos theory to natural and artificial systems. Chaos lies logically between the domains of regular, predictable signals and totally unpredictable *stochastic* signals called *noise*. Chaos is irregular in time and is slightly predictable. It has structure in *phase space*. Chaotic strange

attractors have been found in many environments. Thermal convection in the lower atmosphere was found to be chaotic [18, 15]. Laser fluctuations were found to be chaotic [3, 5, 2], as were the water volume fluctuations in the Great Salt Lake [19]! This signals the necessity of incorporating ideas from chaos theory in the prediction and analysis of natural systems. [1] is a good reference text for the application of chaos in time series prediction.

[1] suggests a method called *False Nearest Neighbours* to determine the required length of the delay coordinate vector. Assuming a delay coordinate length of, say, d, the nearest neighbour (**vnn**) of some vector (**v**) is found and their distance noted. Let the corresponding timestamps be T and TNN. Now the delay coordinate length is increased by 1 (to $d+1$), and the distance between the two vectors at times T and TNN is found. If the distance has increased by a large value, it means that **vnn** was not the actual nearest neighbour to **v** in state space, but had arrived in the neighbourhood of **v** by *projection* from a higher dimension. This implies that the current dimension k was not enough to fully unfold the structure of the system. Such a vector (**vnn**) is called a *False Nearest Neighbour*, and its presence indicates that the proper dimension of the system is greater than the dimension being used currently.

This suggests a technique for finding the "right" length of the delay-coordinate vector. Starting with a low dimension, we calculate the total number of false nearest neighbours for all vectors formed by the time series. As dimension increases, the number of false nearest neighbours should fall off till it reaches some constant value (which might not be zero because of noise). The minimum dimension at which this number stops falling significantly is a good estimate for the length of the delay-coordinate vector. I plan to investigate the practical usefulness of this technique in my work.

3.2.2 Using Fractals

The fractal dimension of the time series gives an estimate of the intrinsic dimensionality of the data. For example, if we consider points along the diagonal of a cube, the intrinsic dimensionality of these points is one (because they actually lie along a straight line), but the extrinsic dimensionality is three (because the individual points are expressed in 3D-coordinates). If the time series is self-similar, that is, the series looks similar whether we look at a short portion of the series or a long portion, the fractal dimension of the time series can be used for prediction. Several time series, such as those for disk-accesses and network topology [8], have been found to obey this assumption.

The idea is to plot fractal dimension of the vector space versus lag length. Beyond a certain point, increasing the lag length adds no new information regarding the state space (by Takens Theorem [22]), so the fractal dimension should flatten out. The lag at which it settles could be used as the optimal lag length. Also, if the fractal dimension increases "significantly" as the lag length is increased from d to $d+1$, we can infer that the $d+1$th component of the delay vector is important. We could then only retain

the important components and discard the others, leading to shorter lag vectors and thus, reducing dimensionality problems. This is analogous to the "feature selection" approach used in tackling several machine learning problems. I still have to figure out how to tag an increase in fractal dimension as "significant". Current algorithms for finding fractal dimension scale almost linearly with number of datapoints, so using this method can lead to increased speed.

3.2.3 Database Issues

Finally, this system depends on making predictions on the basis of past regularities in the data. Thus, the data must be stored in a form making such similarity searches fast and efficient. I have already implemented a time series similarity search prototype, which uses a spatial data access method called the R-Tree [11]. The delay vectors of length d are considered as (infinitesimal) volumes in d-dimensional space, and are recursively grouped together based on their proximity. A height-balanced tree is created where each non-leaf node is associated with a specific volume, and its subtrees may contain points only from that volume. R-Trees allow us to do fast searches over the delay vector space. One problem with this method is the dimensionality curse: the larger the dimension of the delay coordinate vector, the more acute the problem. The dimensionality curse is tackled by the use of certain tricks, as detailed in [7]. A formal method is needed to set the number of nearest neighbours to be used for prediction purposes, perhaps by cross-validation.

Thus, the main aim of my work is to find formal methods for choosing values for delay-vector length, number of nearest neighbours used for interpolation and selecting the important components of the delay-vector. The methods outlined above should work when the data is generated by some chaotic process, such as, perhaps, some physical phenomenon. Also, signal-to-noise ratio should not be too small. I hope to test the methods on several datasets to find conditions under which they work best. The success of the work can be judged by comparing the results produced by using these parameters against those when the parameters are chosen on an ad hoc basis. I also plan to compare the results to those produced by an ARMA model. For this purpose, I intend to use the California-Transportation dataset, which I already have. This contains data on traffic loads on a California highway over time (see Figure 1). If I can get some larger dataset, I also wish to try these techniques on it.

3.3 My Previous Work

I have worked in this area for one term and have already made some progress. A time series similarity-search prototype is already active, and can be accessed on the Internet at the URL http://snapdragon.cald.cs.cmu.edu/TSP. Currently, predictions can be performed on stock price data and California-city transportation data. Screenshots from the system are shown in Fig 2. The length of the delay coordinate vector is chosen on an

ad-hoc basis currently. The dimensionality curse is handled as in [7]. The main objectives for this project is to use the methods outlined in the previous paragraphs to find "right" estimates for the delay vector length and the number of nearest neighbours to be used for prediction. Another aim is to scale up the system to very large datasets (in the order of Gigabytes of data). The proposed research would be the logical extension to the existing system.

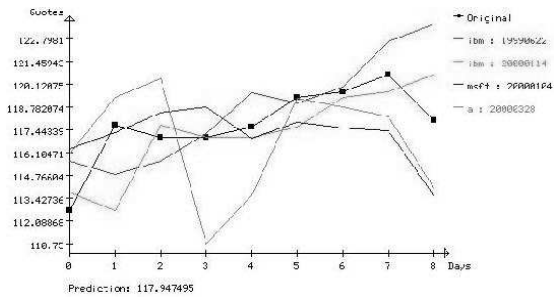


Figure 2: *The current system*: The shows the input values , the 4 nearest neighbours retrieved, and the predicted value.

4 Work Plan

Work	**Date**
False Nearest Neighbours	02/20
Fractal Dimensions	03/01
Delay-coordinate component selection	03/15
Choosing best number of nearest neighbours	04/01
Scalability for large databases	04/15

5 Summary

Time series turn up in a variety of systems, and their analysis and prediction allows us to better understand the systems' internal functioning. My approach involves the use of a technique called *Delay Coordinate Embedding*, under which the time series is broken up into chunks, which give an indication of the internal state of the system. But current techniques for using Delay Coordinate Embedding involve either setting parameters manually, or using "magic" numbers for thresholding, bucketization, etc. This leads to arbitrariness and ad-hoc parameter setting. The aim of my project is to develop techniques to formalize methods for parameter selection and optimization based on the data itself, and also to find methods for efficient storage and retrieval of chunks. This will lead to a method of fast and efficient forecasting of a variety of time sequences.

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